



# CALCULATION PROGRESSION Y1 – Y6

**Date: July 2020**

## Introduction

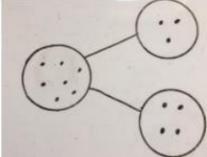
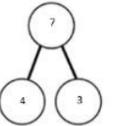
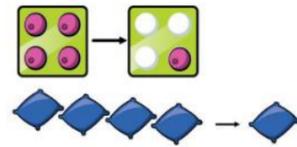
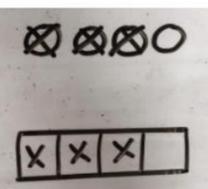
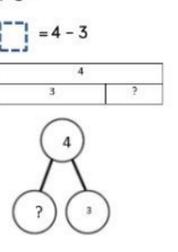
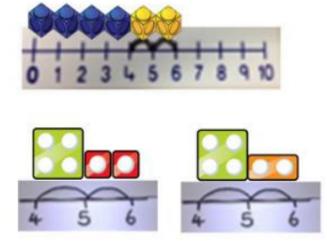
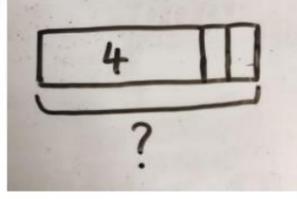
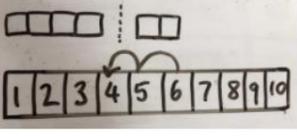
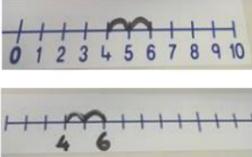
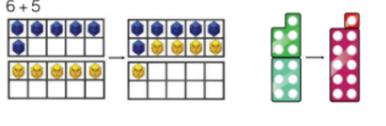
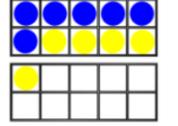
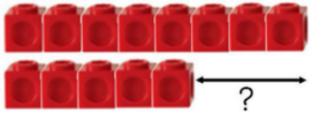
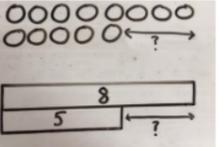
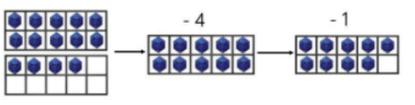
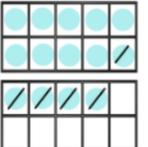
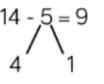
This document lists National Curriculum objectives along with the key concepts and skills, vocabulary and suggested representations to use when teaching calculation. It also gives the relevant objectives from the Lowfield Key Instant Recall Facts (KIRF) grid. These should be used alongside the National Curriculum objectives when planning calculation sessions.

The document is in two main parts: **Addition & Subtraction** and **Multiplication & Division**. Each year group from Y1 – Y6 has its own section within these but the document should ideally be looked at as a whole so that progression is fully understood. There is also a **Formal Methods Layout** appendix.

Y1

NC objectives	KIRF objectives	Concepts / skills	Vocabulary
<p>Ma1/2.2a read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs</p> <p>Ma1/2.2b represent and use number bonds and related subtraction facts within 20</p> <p>Ma1/2.2c add and subtract one-digit and two-digit numbers to 20, including 0</p> <p>Ma1/2.2d solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as <math>7 = ? - 9</math>.</p>	<p>Know all the number bonds to 5.</p> <p>Know all number bonds to 10.</p> <p>Know all addition and subtraction facts for numbers between 0 and 10.</p> <p>Know doubles of all numbers to 10 and halves of even numbers to 10.</p> <p>Know halves of even numbers to 20.</p>	<p>Combining two parts to make a whole: aggregation.</p> <p>Starting with a number and counting on: augmentation.</p> <p>Regrouping / exchanging ones to make ten.</p> <p>Taking away ones by counting back.</p> <p>Finding a difference by comparing two groups.</p>	<p>add, more, plus, and, make, altogether, total, double</p> <p>take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, how many left? difference</p> <p>exchange, regroup, equal to, equals, double, half</p>

### Representations

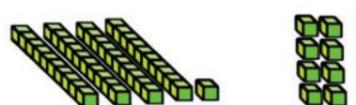
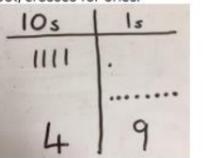
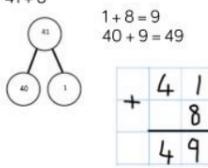
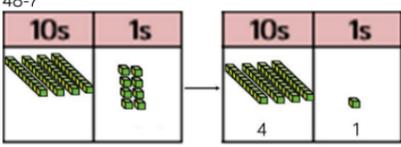
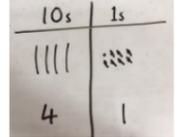
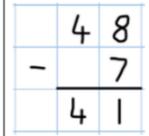
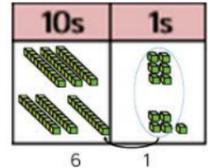
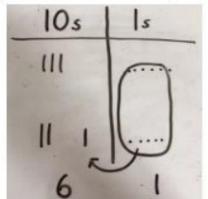
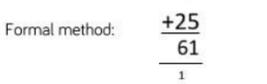
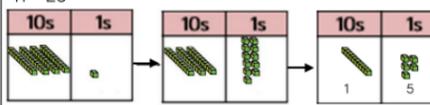
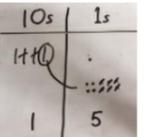
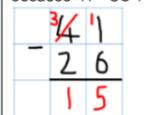
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p><math>4 + 3 = 7</math> Four is a part, 3 is a part and the whole is seven.</p> 	<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p><math>4 - 3 = 1</math></p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p><math>4 - 3 =</math></p> 
<p>Counting on using number lines using cubes or Numicon.</p> 	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? <math>4 + 2</math></p> 	<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p><math>6 - 2 = 4</math></p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p> 
<p>Regrouping to make 10; using ten frames and counters/cubes or using Numicon.</p> <p><math>6 + 5</math></p> 	<p>Children to draw the ten frame and counters/cubes.</p> 	<p>Children to develop an understanding of equality e.g.</p> <p><math>6 + \square = 11</math> <math>6 + 5 = 5 + \square</math> <math>6 + 5 = \square + 4</math></p>	<p>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).</p> <p>Calculate the difference between 8 and 5.</p> 	<p>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.</p> 	<p>Find the difference between 8 and 5.</p> <p><math>8 - 5</math>, the difference is <input type="text"/></p> <p>Children to explore why <math>9 - 6 = 8 - 5 = 7 - 4</math> have the same difference.</p>
<p>Making 10 using ten frames.</p> <p><math>14 - 5</math></p> 	<p>Children to present the ten frame pictorially and discuss what they did to make 10.</p> 	<p>Children to show how they can make 10 by partitioning the subtrahend.</p> <p><math>14 - 5 = 9</math></p>  <p><math>14 - 4 = 10</math> <math>10 - 1 = 9</math></p>			

Y2

NC objectives	KIRF objectives	Concepts / skills	Vocabulary
<p>Ma2/2.2a solve problems with addition and subtraction:</p> <ul style="list-style-type: none"> <li>i. using concrete objects and pictorial representations, including those involving numbers, quantities and measures</li> <li>ii. applying their increasing knowledge of mental and written methods</li> </ul> <p>Ma2/2.2b recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</p> <p>Ma2/2.2c add and subtract numbers using concrete objects, pictorial representations, and mentally, including:</p> <ul style="list-style-type: none"> <li>i. a two-digit number and 1s</li> <li>ii. a two-digit number and 10s</li> <li>iii. 2 two-digit numbers</li> <li>iv. adding 3 one-digit numbers</li> </ul> <p>Ma2/2.2d show that addition of 2 numbers can be done in any order (commutative) and subtraction of one number from another cannot</p> <p>Ma2/2.2e recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</p>	<p>Know all addition and subtraction facts for numbers between 11 and 20.</p> <p>Know all addition and subtraction facts using multiples of 10 to 100.</p>	<p>Adding three single digits by adding two first, e.g. to make 10, and then adding the third.</p> <p>Using Base 10 to 'make' and then combine two numbers, including regrouping / exchanging ones to make ten.</p> <p>Using knowledge of number bonds within 10 to support more complex calculations.</p> <p>Using Base 10 to 'take' one number from another, including exchanging tens for ones.</p> <p>Finding the difference by counting on and by subtracting.</p>	<p>(New vocabulary in <b>bold</b>)</p> <p>add, more, plus, and, make, altogether, total, double, <b>tens, ones, partition, addition, column</b></p> <p>take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, how many left, difference, <b>how much less is_?</b></p> <p>exchange, regroup, equal to, equals, double, half, <b>inverse, strategy</b></p>

**Representations**

NB refer to Formal Methods Layout appendix – examples here are suggestions only.

<p>TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8</p> 	<p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p> 	<p>41 + 8</p> <p>1 + 8 = 9 40 + 9 = 49</p> 	<p>Column method using base 10. 48 - 7</p> 	<p>Children to represent the base 10 pictorially.</p> 	<p>Column method or children could count back 7.</p> 
<p>TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25</p> 	<p>Children to represent the base 10 in a place value chart.</p> 	<p>Looking for ways to make 10.</p> <p>36 + 25 = 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61</p> <p>Formal method:</p> 	<p>Column method using base 10 and having to exchange. 41 - 26</p> 	<p>Represent the base 10 pictorially, remembering to show the exchange.</p> 	<p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.</p> 

Y3

NC objectives	KIRF objectives	Concepts / skills	Vocabulary
<p>Ma3/2.2a add and subtract numbers mentally, including:</p> <ul style="list-style-type: none"> <li>i. a three-digit number and 1s</li> <li>ii. a three-digit number and 10s</li> <li>iii. a three-digit number and 100s</li> </ul> <p>Ma3/2.2b add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction</p> <p>Ma3/2.2c estimate the answer to a calculation and use inverse operations to check answers</p>	<p>Revise all the addition and subtraction facts to 20.</p> <p>Know doubles of all whole numbers to 25 &amp; all multiples of 10 to 250.</p> <p>Know halves of all even numbers to 50 &amp; all multiples of 10 to 500.</p> <p>Know all addition and subtraction facts using:</p> <ul style="list-style-type: none"> <li>• multiples of 100 to 1,000</li> <li>• multiples of 5 to 100</li> </ul>	<p>Column addition method with exchange / regrouping.</p> <p>Column subtraction method with exchange.</p> <p>Using place value counters (up to 3 digits).</p>	<p>(New vocabulary in <b>bold</b>)</p> <p>add, more, plus, and, make, altogether, total, double, tens, ones, partition, addition, column, <b>addend</b>, <b>sum</b></p> <p>take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, how many left, how much less is_? difference, <b>minuend</b>, <b>subtrahend</b></p> <p>exchange, regroup, equal to, equals, double, half, inverse, strategy, <b>regroup</b>, <b>exchange</b>, <b>estimate</b></p>

**Representations**

NB refer to Formal Methods Layout appendix – examples here are suggestions only.

<p><b>Use of place value counters to add HTO + TO, HTO + HTO etc.</b> When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.</p>	<p>Children to represent the counters in a place value chart, circling when they make an exchange.</p>	$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 11 \end{array}$	<p><b>Column method</b> using place value counters.</p> <p>234 - 88</p>	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p>	<p>Formal column method. Children must understand what has happened when they have crossed out digits.</p> $\begin{array}{r} 234 \\ - 88 \\ \hline 146 \end{array}$									
<p><b>Conceptual variation; different ways to ask children to solve 21 + 34</b></p>														
	<p>Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?</p> <p>21 + 34 = 55. Prove it</p>	$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$ <p><b>21 + 34 =</b></p> <p><math>\square = 21 + 34</math></p> <p>Calculate the sum of twenty-one and thirty-four.</p>	<p>Missing digit problems:</p> <table border="1"> <tr> <td>10s</td> <td>1s</td> </tr> <tr> <td>2</td> <td>1</td> </tr> <tr> <td>3</td> <td>?</td> </tr> <tr> <td>?</td> <td>5</td> </tr> </table>	10s	1s	2	1	3	?	?	5	<p><b>Conceptual variation; different ways to ask children to solve 391 - 186</b></p>	<p>Raj spent £391, Timmy spent £186. How much more did Raj spend?</p> <p>Calculate the difference between 391 and 186.</p> <p><math>\square = 391 - 186</math></p> $\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$ <p>What is 186 less than 391?</p>	<p>Missing digit calculations</p> $\begin{array}{r} 39\square \\ - \square\square 6 \\ \hline \square 0 5 \end{array}$
10s	1s													
2	1													
3	?													
?	5													

	NC objectives	KIRF objectives	Concepts / skills	Vocabulary
Addition and Subtraction	<p>Ma4/2.2a add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate</p> <p>Ma4/2.2b estimate and use inverse operations to check answers to a calculation</p> <p>Ma4/3.1d estimate, compare and calculate different measures, including money in pounds and pence</p>	<p>Know all 2-digit pairs that total 100.</p> <p>Know doubles of:</p> <ul style="list-style-type: none"> <li>all whole numbers to 50</li> <li>all multiples of 50 to 500</li> </ul> <p>Know halves of:</p> <ul style="list-style-type: none"> <li>all whole numbers to 100</li> <li>all multiples of 50 to 1,000</li> </ul> <p>Know all pairs of multiples of 50 with a total of 1,000.</p>	<p>Column addition method with exchange / regrouping.</p> <p>Column subtraction method with exchange.</p> <p>Using decimal notation for calculations set in a practical context, i.e. money.</p>	<p>add, more, plus, and, make, altogether, total, double, tens, ones, partition, addition, column, addend, sum</p> <p>take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, how many left, how much less is_? difference, minuend, subtrahend</p> <p>exchange, regroup, equal to, equals, double, half, inverse, strategy, regroup, exchange, estimate, <b>decimal</b></p>
	<p><b>Representations</b></p> <p>Pupils should now normally use the formal column method for both addition and subtraction (see Formal Methods Layout appendix).</p> <p>For consolidation or to address misconceptions, appropriate representations from earlier year groups should be revisited.</p>			

	NC objectives	KIRF objectives	Concepts / skills	Vocabulary
Addition and Subtraction	<p>Ma5/2.2a add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)</p> <p>Ma5/2.2b add and subtract numbers mentally with increasingly large numbers</p> <p>Ma5/2.2c use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy</p> <p><i>*Ma5/3.1g use all four operations to solve problems involving measure using decimal notation including scaling</i></p>	<p>Know all decimal pairs that total 1 or 10 (to 1 decimal place) – <i>link to Measurement*</i></p> <p>Know the doubles and halves of all two-digit numbers.</p> <p>Know doubles and halves of all multiples of 10 to 1,000.</p>	<p>Column addition method with exchange / regrouping.</p> <p>Use of place value counters for adding decimals.</p> <p>Column subtraction method with exchange.</p> <p>Use of place value counters for subtracting decimals with the same number of decimal places.</p>	<p>add, more, plus, and, make, altogether, total, double, tens, ones, partition, addition, column, addend, sum</p> <p>take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, how many left, how much less is_? difference, minuend, subtrahend</p> <p>exchange, regroup, equal to, equals, double, half, inverse, strategy, regroup, exchange, estimate</p>
	<p><b>Representations</b></p> <p>Pupils should now normally use the formal column method for both addition and subtraction (see Formal Methods Layout appendix).</p> <p>For consolidation or to address misconceptions, appropriate representations from earlier year groups should be revisited.</p>			

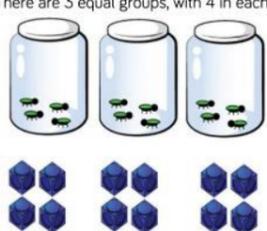
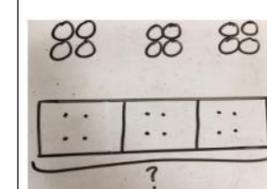
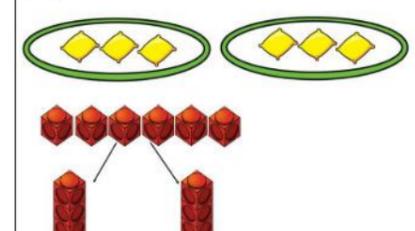
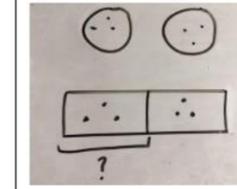
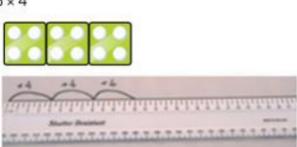
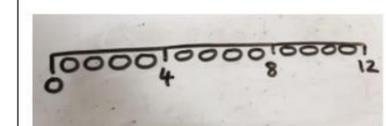
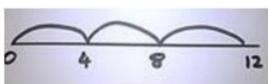
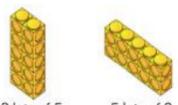
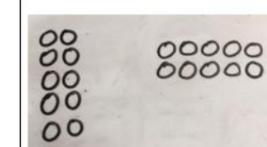
For guidance on teaching addition and subtraction to Y6, please see the combined Y6 Addition, Subtraction, Multiplication and Division section below.

Y1

NC objectives	KIRF objectives	Concepts / skills	Vocabulary
<p>Ma1/2.1b count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s</p> <p>Ma1/2.3a solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p>	<p>Count forward to 100 in steps of 2, 5 and 10.</p> <p>Know doubles of all numbers to 10 and halves of even numbers to 10.</p> <p>Know halves of even numbers to 20.</p>	<p>Recognising and making equal groups.</p> <p>Doubling and halving.</p> <p>Counting in multiples.</p> <p>Division by sharing objects equally into a given number of groups, e.g. I have 12 sweets and share them equally into 3 groups - how many in each group? (Partitive division.)</p> <p>Division as making equal groups of a given size, e.g. I have 12 sweets and put them in groups of 3, how many groups? (Quotative division.)</p>	<p>groups of, lots of, times, array, altogether, multiply, repeated addition, count in...</p> <p>group, share, share equally, one each, two each...</p>

### Representations

NB when **multiplying**, the number of groups or the group sizes should usually be based around 2, 5 or 10. These examples are for illustration only.

<p><b>Repeated grouping/repeated addition</b>  <math>3 \times 4</math>  <math>4 + 4 + 4</math>                      There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p><math>3 \times 4 = 12</math>  <math>4 + 4 + 4 = 12</math></p>	<p><b>Sharing using a range of objects.</b>  <math>6 \div 2</math></p> 	<p>Represent the sharing pictorially.</p> 	<p><math>6 \div 2 = 3</math></p> <table border="1" data-bbox="2404 1176 2671 1239"> <tr> <td>3</td> <td>3</td> </tr> </table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3						
<p><b>Number lines to show repeated groups-</b>  <math>3 \times 4</math></p>  <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jumps of four.</p> <p><math>3 \times 4 = 12</math></p> 					
<p><b>Use arrays to illustrate commutativity</b> counters and other objects can also be used.  <math>2 \times 5 = 5 \times 2</math></p>  <p>2 lots of 5      5 lots of 2</p>	<p>Children to represent the arrays pictorially.</p> 	<p>Children to be able to use an array to write a range of calculations e.g.</p> <p><math>10 = 2 \times 5</math>  <math>5 \times 2 = 10</math>  <math>2 + 2 + 2 + 2 + 2 = 10</math>  <math>10 = 5 + 5</math></p>					

Y2

NC objectives	KIRF objectives	Concepts / skills	Vocabulary
<p>Ma2/2.1a count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward</p> <p>Ma2/2.3a recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers</p> <p>Ma2/2.3b calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (<math>\times</math>), division (<math>\div</math>) and equals (=) signs</p> <p>Ma2/2.3c show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot</p> <p>Ma2/2.3d solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.</p>	<p>Know multiplication and division facts for 2x table. *up to <math>12 \times 2</math></p> <p>Know multiplication and division facts for 10x table. *up to <math>12 \times 10</math></p> <p>Know multiplication and division facts for 5x table. *up to <math>12 \times 5</math></p> <p>Know all multiplication and division facts for 3x table. *up to <math>12 \times 3</math></p>	<p>Showing the commutative quality of multiplication, e.g. using arrays to show that <math>3 \times 5 = 15</math> and <math>5 \times 3 = 15</math></p> <p>Linking division to multiplication, e.g. by finding division facts within arrays.</p> <p>Repeated subtraction, e.g. by using a number line.</p>	<p>(New vocabulary in <b>bold</b>)</p> <p>groups of, lots of, times, array, altogether, multiply, count in, <b>repeated addition, multiplied by, commutative, sets of, equal groups, times as big as, once, twice, three times...</b></p> <p><b>column, row</b> (in arrays)</p> <p>group, share, share equally, one each, two each..., <b>divide, divided by, divided into, division, left, left over</b></p>

**Representations**

Use arrays to illustrate commutativity counters and other objects can also be used.  
 $2 \times 5 = 5 \times 2$

Children to represent the arrays pictorially.

Children to be able to use an array to write a range of calculations e.g.

$10 = 2 \times 5$   
 $5 \times 2 = 10$   
 $2 + 2 + 2 + 2 + 2 = 10$   
 $10 = 5 + 5$

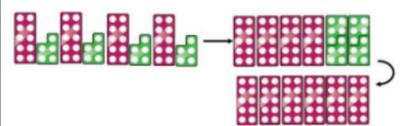
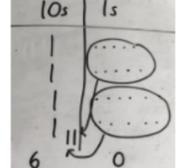
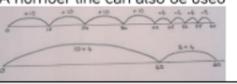
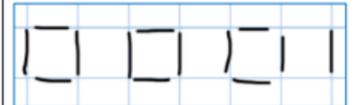
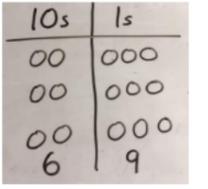
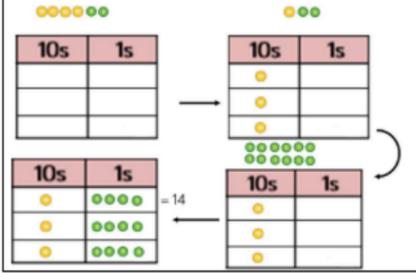
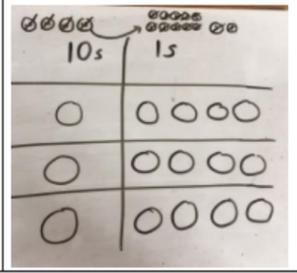
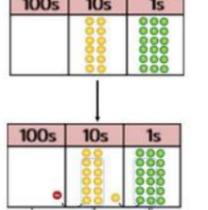
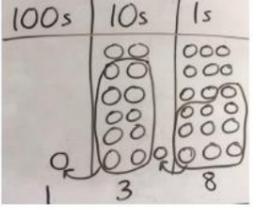
Repeated subtraction using Cuisenaire rods above a ruler.  
 $6 \div 2$

3 groups of 2

Children to represent repeated subtraction pictorially.

Abstract number line to represent the equal groups that have been subtracted.

Y3

	NC objectives	KIRF objectives	Concepts / skills	Vocabulary		
	<p>Ma3/2.1a count from 0 in multiples of 4, 8, 50 and 100</p> <p>Ma3/2.3a recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables</p> <p>Ma3/2.3b write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods</p>	<p>Revise multiplication and division facts for 2x, 3x, 5x and 10x tables.</p> <p>Know all multiplication and division facts for and 4x table. *up to 12 x 4</p> <p>Know multiplication and division facts for 8x table. *up to 12 x 8</p> <p>Know doubles of all whole numbers to 25 &amp; all multiples of 10 to 250</p> <p>Know halves of all even numbers to 50 &amp; all multiples of 10 to 500</p>	<p>2 digit x 1 digit whole numbers, e.g. shown with Base 10.</p> <p>Division with a remainder, using times tables facts or repeated subtraction, e.g. shown with lollipop sticks.</p> <p>2 digit ÷ 1 digit whole numbers, e.g. shown with Base 10 or place value counters.</p>	<p>(New vocabulary in <b>bold</b>)</p> <p>groups of, lots of, times, array, altogether, multiply, count in, repeated addition, multiplied by, commutative, sets of, equal groups, times as big as, once, twice, three times..., <b>partition, multiple, product, tens, ones</b></p> <p>column, row (<i>in arrays</i>)</p> <p>group, share, share equally, one each, two each..., divide, divided by, divided into, division, left, left over, <b>inverse, remainder</b></p>		
	<p><b>Representations</b></p> <p>NB refer to Formal Methods Layout appendix – examples here are suggestions only.</p>					
	<p>Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 x 15</p> 	<p>Children to represent the concrete manipulatives pictorially.</p> 	<p>Children to be encouraged to show the steps they have taken.</p> $\begin{array}{r} 4 \times 15 \\ 10 \quad 5 \\ \hline 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$ <p>A number line can also be used</p> 	<p>2d + 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4</p> <p>Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.</p>  <p>There are 3 whole squares, with 1 left over.</p>	<p>Children to represent the lollipop sticks pictorially.</p>  <p>There are 3 whole squares, with 1 left over.</p>	<p>13 ÷ 4 = 3 remainder 1</p> <p>Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.</p> <p>'3 groups of 4, with 1 left over'</p> 
	<p>Formal column method with place value counters (base 10 can also be used.) 3 x 23</p> 	<p>Children to represent the counters pictorially.</p> 	<p>Children to record what it is they are doing to show understanding.</p> $\begin{array}{r} 3 \times 23 \\ 20 \quad 3 \\ \hline 60 + 9 = 69 \end{array}$ $\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$	<p>Sharing using place value counters. 42 ÷ 3 = 14</p> 	<p>Children to represent the place value counters pictorially.</p> 	<p>Children to be able to make sense of the place value counters and write calculations to show the process.</p> $\begin{array}{r} 42 \div 3 \\ 42 = 30 + 12 \\ 30 \div 3 = 10 \\ 12 \div 3 = 4 \\ 10 + 4 = 14 \end{array}$
	<p>Formal column method with place value counters. 6 x 23</p> 	<p>Children to represent the counters/base 10, pictorially e.g. the image below.</p> 	<p>Formal written method</p> $\begin{array}{r} 6 \times 23 = \\ 23 \\ \times 6 \\ \hline 138 \\ 1 \quad 1 \end{array}$			

Y4

	NC objectives	KIRF objectives	Concepts / skills	Vocabulary
	<p>Ma4/2.1a count in multiples of 6, 7, 9, 25 and 1,000</p> <p>Ma4/2.3a recall multiplication and division facts for multiplication tables up to <math>12 \times 12</math></p> <p>Ma4/2.3b use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers</p> <p>Ma4/2.3c recognise and use factor pairs and commutativity in mental calculations</p> <p>Ma4/2.3d multiply two-digit and three-digit numbers by a one-digit number using formal written layout</p>	<p>Know multiplication and division facts for the 6x table. *up to <math>12 \times 6</math></p> <p>Know multiplication and division facts for the 7x and 9x tables. *up to <math>12 \times 7</math> &amp; <math>12 \times 9</math></p> <p>Know multiplication and division facts for the 11x table. *up to <math>12 \times 11</math></p> <p>Know all multiplication and division facts for all tables up to <math>12 \times 12</math>.</p> <p>Know doubles of:</p> <ul style="list-style-type: none"> <li>all whole numbers to 50</li> <li>all multiples of 50 to 500</li> </ul> <p>Know halves of:</p> <ul style="list-style-type: none"> <li>all whole numbers to 100</li> <li>all multiples of 50 to 1,000</li> </ul>	<p>Using column multiplication supported by place value counters (2 and 3 digit x 1 digit).</p> <p>Using the short division method (up to 3 digits <math>\div</math> 1 digit) through concrete and pictorial representations.</p>	<p>(New vocabulary in <b>bold</b>)</p> <p>groups of, lots of, times, array, altogether, multiply, count in, repeated addition, multiplied by, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, multiple, product, tens, ones, <b>factor</b></p> <p>column, row (<i>in arrays</i>)</p> <p>group, share, share equally, one each, two each..., divide, divided by, divided into, division, left, left over, inverse, remainder, multiple, <b>short division, divisible by</b></p>

### Representations

NB refer to Formal Methods Layout appendix – examples here are suggestions only.

Formal column method with place value counters.  
 $6 \times 23$

Children to represent the counters/base 10, pictorially e.g. the image below.

Formal written method

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 11
 \end{array}$$

Short division using place value counters to group.  
 $615 \div 5$

1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

Further representations can be used to make links where needed, e.g. expanded methods for written multiplication.

Y5

	NC objectives	KIRF objectives	Concepts / skills	Vocabulary
	<p>Ma5/2.3a identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers</p> <p>Ma5/2.3d multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers</p> <p>Ma5/2.3e multiply and divide numbers mentally drawing upon known facts</p> <p>Ma5/2.3f divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</p> <p>Ma5/2.3h recognise and use square numbers and cube numbers, and the notation for squared (<sup>2</sup>) and cubed (<sup>3</sup>)</p> <p>Ma5/3.1g use all four operations to solve problems involving measure using decimal notation including scaling</p>	<p>Revise multiplication and division facts for all times tables up to 12 x 12.</p> <p>Know the doubles and halves of all two-digit numbers.</p> <p>Know doubles and halves of all multiples of 10 to 1,000.</p> <p>Know all pairs of factors of numbers up to 100.</p> <p>Know square numbers to 12 x 12.</p>	<p>Using formal written methods to multiply (up to 4 digit x 1 or 2 digit numbers).</p> <p>Using the written short division method (up to 4 digits ÷ 1 digit).</p>	<p>(New vocabulary in <b>bold</b>)</p> <p>groups of, lots of, times, array, altogether, multiply, count in, repeated addition, multiplied by, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, multiple, product, tens, ones, factor, <b>square, cube, integer</b></p> <p>column, row (<i>in arrays</i>), <b>decimal</b></p> <p>group, share, share equally, one each, two each..., divide, divided by, divided into, division, left, left over, inverse, remainder, multiple, short division, divisible by, <b>dividend, divisor, quotient</b></p>

**Representations**

NB also refer to Formal Methods Layout appendix.

$$\begin{array}{r}
 342 \\
 \times \quad 7 \\
 \hline
 2,394 \\
 \hline
 21
 \end{array}$$

$$\begin{array}{r}
 1234 \\
 \times \quad 16 \\
 \hline
 7404 \\
 12340 \\
 \hline
 19744
 \end{array}$$

$$\begin{array}{r}
 123 \\
 5 \overline{)615}
 \end{array}$$

$$\begin{array}{r}
 0663r5 \\
 8 \overline{)5309}
 \end{array}$$

For consolidation or to address misconceptions, appropriate representations from earlier year groups should be revisited. Expanded written methods can also be used to make links when required.

Y6

NC objectives	KIRF objectives	Concepts / skills	Vocabulary
<p>Ma6/2.2a multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</p> <p>Ma6/2.2b divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</p> <p>Ma6/2.2c divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context</p> <p>Ma6/2.2d perform mental calculations, including with mixed operations and large numbers.</p> <p>Ma6/2.2f use their knowledge of the order of operations to carry out calculations involving the 4 operations</p> <p>Ma6/2.2i use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.</p> <p>Ma6/2.3h multiply one-digit numbers with up to 2 decimal places by whole numbers</p> <p>Ma6/2.3i use written division methods in cases where the answer has up to 2 decimal places</p> <p>Ma6/3.1a solve problems involving the calculation and conversion of units of measure, using decimal notation up to 2 decimal places where appropriate</p>	<p>Know the two-place decimal complements of 1.</p> <p>Use all multiplication and division facts for times tables to <math>12 \times 12</math> to derive <math>\times</math> and <math>\div</math> of small multiples of 10 and 100 (e.g. <math>30 \times 900</math>; <math>8,100 \div 9</math>).</p> <p>Know the decimal and percentage equivalents of the fractions <math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{3}{4}</math>, <math>\frac{1}{3}</math>, <math>\frac{2}{3}</math>, tenths and fifths.</p> <p>Know the doubles and halves of all multiples of 100 to 10,000.</p>	<p>Column addition method with exchange / regrouping.</p> <p>Use of place value counters for adding decimals.</p> <p>Column subtraction method with exchange.</p> <p>Use of place value counters for subtracting decimals with different numbers of decimal places.</p> <p>Using formal written methods to multiply (up to 4 digit <math>\times</math> 1 or 2 digit numbers).</p> <p>Using the written short division method (up to 4 digits <math>\div</math> 1 digit).</p> <p>Long division and short division (up to 4 digits <math>\div</math> 2 digits) supported by place value counters, including exchange into the tenths and hundredths columns.</p>	<p>add, more, plus, and, make, altogether, total, double, tens, ones, partition, addition, column, addend, sum</p> <p>take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, how many left, how much less is_? difference, minuend, subtrahend</p> <p>equal to, equals, double, half, inverse, strategy, regroup, exchange, estimate</p> <p>groups of, lots of, times, array, altogether, multiply, count in, repeated addition, multiplied by, commutative, sets of, equal groups, times as big as, once, twice, three times..., partition, multiple, product, tens, ones, factor, square, cube, integer</p> <p>column, row (<i>in arrays</i>), decimal</p> <p>group, share, share equally, one each, two each..., divide, divided by, divided into, division, left, left over, inverse, remainder, multiple, short division, divisible by, dividend, divisor, quotient</p>

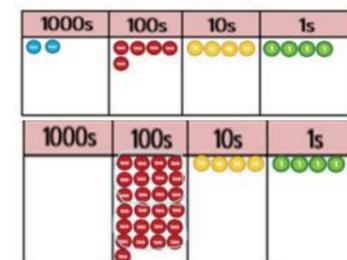
**Representations** NB also refer to Formal Methods Layout appendix.

Pupils should now normally use formal written methods for multiplication and short division with a 1 digit divisor. Written methods for long and short division with a 2 digit divisor should be supported by other representations (see examples below).

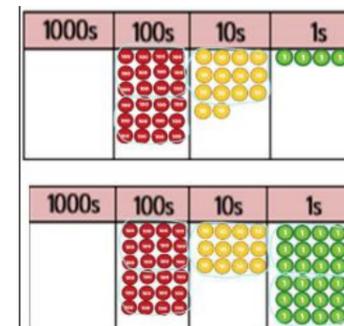
Pupils should now normally use the formal column method for both addition and subtraction.

For consolidation or to address misconceptions, appropriate representations from earlier year groups should be revisited.

Long division using place value counters  
 $2544 \div 12$



$$\begin{array}{r} 02 \\ 12 \overline{)2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

## Appendix: Formal Methods Layout

This table shows how calculations should be set out once pupils have mastered the underlying concepts and are ready to use more efficient abstract methods of recording. Concrete and pictorial representations, or expanded written methods, should be used until pupils are secure enough in their understanding to attempt these methods.

<b>Addition</b>	$  \begin{array}{r}  789 \\  + 642 \\  \hline  1431 \\  \hline  11  \end{array}  $	<b>Multiplication</b>	$  \begin{array}{r}  342 \\  \times 7 \\  \hline  2394 \\  \hline  21  \end{array}  $	$  \begin{array}{r}  12 \\  124 \\  \times 26 \\  \hline  744 \\  2480 \\  \hline  3224 \\  \hline  11  \end{array}  $
<b>Subtraction</b>	$  \begin{array}{r}  8 \quad 12 \quad 1 \\  932 \\  - 457 \\  \hline  475 \\  \hline  \end{array}  $	<b>Division</b>	$  \begin{array}{r}  86 \text{ r}2 \\  5 \overline{) 432} \\  \underline{20} \phantom{0} \\  23 \phantom{0} \\  \underline{15} \phantom{0} \\  8 \phantom{0} \\  \underline{5} \phantom{0} \\  3 \phantom{0} \\  \underline{2} \phantom{0} \\  12  \end{array}  $	$  \begin{array}{r}  28 \text{ r}12 \\  15 \overline{) 432} \\  \underline{30} \phantom{0} \\  13 \phantom{0} \\  \underline{12} \phantom{0} \\  12 \phantom{0} \\  \underline{12} \phantom{0} \\  0  \end{array}  $ <p style="text-align: right;">20 x 15</p> <p style="text-align: right;">8 x 15</p>